Pillai College of Engineering.
End Semester Examination.
Branch FE (Mech/Auto)

Subject: Engineering Mathematics I
Max Marks: 60.
Time: 2 hrs.
Date: 4/4/2022.

Q1)

(a) $x^6+1=0.$

$x=(-1)^{1/6}.$

$=(\cos\pi+i\sin\pi)^{1/6}.$

$=\cos(2k+1)\frac{\pi}{6}+i\sin(2k+1)\frac{\pi}{6}.$ ——— (2)

where $k=0,1,2,3,4,5.$

$x_0=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}.$

$x_1=\cos 3\frac{\pi}{6}+i\sin 3\frac{\pi}{6}.$

$x_2=\cos 5\frac{\pi}{6}+i\sin 5\frac{\pi}{6}.$

$x_3=\cos 7\frac{\pi}{6}+i\sin 7\frac{\pi}{6}.$

$x_4=\cos 9\frac{\pi}{6}+i\sin 9\pi/6.$ ——— (3)

$x_5=\cos 11\pi/6+i\sin 11\pi/6.$

(b) $y=\frac{x}{x^2-16}=\frac{x}{(x+4)(x-4)}=\frac{A}{(x+4)}+\frac{B}{(x-4)}$

$\Rightarrow x=A(x-4)+B(x+4)$

$\Rightarrow$ Put $x=4 \Rightarrow B=1/2$.

Put $x=-4 \Rightarrow A=1/2$. ——— (2)

$\therefore y=\frac{1}{2}\cdot\frac{1}{x+4}+\frac{1}{2}\frac{1}{x-4}$

$y_n=\frac{1}{2}\left[\frac{(-1)^n\cdot n!}{(x+4)^{n+1}}+\frac{(-1)^n\cdot n!}{(x-4)^{n+1}}\right]$

If $y=\frac{1}{ax+b}$ then

$y_n=\frac{(-1)^n\cdot n!\,a^n}{(ax+b)^{n+n}}.$

——— (3)

(c) If u is a homogeneous function of two variables of degree n then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n \cdot u$. —— (1)

(i) $u = \log \frac{x}{y} + \log \frac{y}{x}$.

Put $x = xt$, $y = yt$.

$\therefore u = t^0 \left[\log \frac{x}{y} + \log \frac{y}{x}\right]$

$\Rightarrow u$ is homogeneous of degree 0.

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n \cdot u. = 0$$ —— (2)

(ii) $u = \frac{x^3+y^3}{xy}$.

u is hom of degree 1

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$.

—— (2).

(d) $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$C_3 \to C_3 - C_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$C_3 \to C_3 + C_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ —— (4)

Rank $(A) = 3$ —— (1).

Q2)

(a) Maclaurin's series: $f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^n}{n!} f^n(0) + \cdots$

$y = \log(1+x) \rightarrow y(0) = \log(1+0) = \underline{\underline{0}}$.

$y_1 = \frac{1}{1+x} \Rightarrow y_1(0) = \underline{\underline{1}}$.

$y_2 = -\frac{1}{(1+x)^2} \rightarrow y_2(0) = \underline{\underline{-1}}$

$y_3 = \frac{2}{(1+x)^3} \Rightarrow y_3(0) = \underline{\underline{2}}$. —— 2

$\vdots$

$\therefore y = \log(1+x) = 0 + \frac{x}{1!} - \frac{x^2}{2!} + 2 \cdot \frac{x^3}{3!} - 6 \cdot \frac{x^4}{4!} + \cdots$ —— 2

$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

(b) $u = e^x \cos y \qquad v = e^x \sin y$.

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin y & e^x \cos y \end{vmatrix}$$ —— 1

$= e^{2x}[\cos^2 y + \sin^2 y] = \underline{\underline{e^{2x}}}$ —— 3

(c). $x + 2y - z = 1$
$x + y + 2z = 9$.
$2x + y - z = 2$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 1 & 1 & 2 & 9 \\ 2 & 1 & -1 & 2 \end{array}\right]$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$
$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 0 & -1 & 3 & 8 \\ 0 & -3 & 1 & 0 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$
$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 1 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & -8 & -24 \end{array}\right] \text{ --- } 4$$

$\Rightarrow x + 2y - z = 1 \to (1)$

$-y + 3z = 8 \to (2)$

$-8z = -24 \Rightarrow \underline{\underline{z = 3}}$

$(2) \Rightarrow -y = -1 \Rightarrow \underline{\underline{y = 1}}$

$(1) \Rightarrow x + 2 - 3 = 1$

$\Rightarrow \underline{x = 2}$ —— 2.

$\underline{\underline{(2, 1, 3) = (x, y, z)}}$

(d) $\log[\sin(x + iy)] = a + ib$.

$\log[\sin x \cosh y + i \cos x \sinh y] = a + ib$.

$\frac{1}{2}\log[\sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y] + i \tan^{-1} \frac{\cos x \sinh y}{\sin x \cosh y}$

$= a + ib$. —— 2

$\Rightarrow a = \frac{1}{2}\log\left[\left[\frac{1 - \cos 2x}{2}\right]\cosh^2 y + \left(\frac{1 + \cos 2x}{2}\right)(-1 + \cosh^2 y)\right]$

$= \frac{1}{2}\log\left[\cancel{\frac{\cosh^2 y}{2}} - \cancel{\frac{\cos 2x \cosh^2 y}{2}} - \frac{1}{2} - \frac{\cos 2x}{2} + \frac{\cosh^2 y}{2} + \cancel{\frac{\cos 2x \cosh^2 y}{2}}\right]$

$$= \frac{1}{2}\log\left[\frac{2\cosh^2 y}{2} - \frac{\cos 2x}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\log\left[\frac{1+\cosh 2y}{2} - \frac{\cos 2x}{2} - \frac{1}{2}\right]$$

$$a = \frac{1}{2}\log\left[\frac{\cosh 2y - \cos 2x}{2}\right] \quad \text{—— (2)}$$

$$2a = \log \frac{\cosh 2y - \cos 2x}{2}$$

$$\Rightarrow 2e^{2a} = \cosh 2y - \cos 2x.$$

$$b = \tan^{-1}[\cot x . \tanh y].$$

$$\Rightarrow \tan b = \cot x . \tanh y. \quad \text{—— (2)}$$

Q3

a) Orthogonal: $AA^T = A^T A = I.$ —— (1)

$$A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$A' = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

$$AA' = \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly $A'A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ A is orthogonal —— (2)

$$A^{-1} = A' = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} \quad \text{—— (1)}$$

b) $u = f(2x-3y, 3y-4z, 4z-2x)$

$P = 2x-3y, \quad Q = 3y-4z, \quad R = 4z-2x.$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial P}\cdot\frac{\partial P}{\partial x} + \frac{\partial u}{\partial Q}\cdot\frac{\partial Q}{\partial x} + \frac{\partial u}{\partial R}\cdot\frac{\partial R}{\partial x}.$$

$$= \frac{\partial u}{\partial P}(2) + \frac{\partial u}{\partial Q}\cdot(0) + \frac{\partial u}{\partial R}\cdot(-2)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial P}\cdot(-3) + \frac{\partial u}{\partial Q}(3) + \frac{\partial u}{\partial R}(0)$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial P}(0) + \frac{\partial u}{\partial Q}(-4) + \frac{\partial u}{\partial R}\cdot(4) \quad \text{--- } 2$$

$$\therefore 6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 12\frac{\partial u}{\partial P} - 12\frac{\partial u}{\partial R} - 12\frac{\partial u}{\partial P} + 12\frac{\partial u}{\partial Q} - 12\frac{\partial u}{\partial Q} + 12\frac{\partial u}{\partial R}$$

$$= 0 \quad \text{--- } 2.$$

c) $15x + y + z = 17$

$2x + 15y + z = 18$

$x + 2y + 15z = 18.$

$$\therefore \quad x = \frac{1}{15}[17 - y - z]$$

$$y = \frac{1}{15}[18 - 2x - z]$$

$$z = \frac{1}{15}[18 - x - 2y] \quad \text{--- } 1$$

First iteration

$x_1 = 1.1333 \quad [y = 0, z = 0]$

$y_1 = 1.0489 \quad [x = 1.1333, z = 0]$

$z_1 = 0.9090. \quad [x = 1.1333, y = 1.0489] \quad \text{--- } 1$

Second iteration

$y = 1.0489,\ z = 0.9090$

$x_2 = 1.0028.$

$x = 1.0028,\ z = 0.9090$

$y_2 = 1.0057$

$x = 1.0028,\ y = 1.0057$ — 1

$z_2 = 0.9991$

Third iteration

$y = 1.0057,\ z = 0.9991$

$x_3 = 0.9997$

$x = 0.9997,\ z = 0.9991$

$y_3 = 1.0001$

$x = 0.9997,\ y = 1.0001$

$z_3 = 1.0000$ — 1

Fourth iteration

$y = 1.0001,\ z = 1.0000.$

$x_4 = 0.9999$

$x = 0.9999,\ z = 1.0000$

$y_4 = 1.0000$

$x = 0.9999,\ y = 1.0000.$

$z_4 = 1.0000.$ — 2

Roots $x = 1,\ y = 1,\ z = 1$

d)

X	Y	$x = X - A$	$y = Y - B$	x^2	y^2	xy
25	70	-12	5	144	25	-60
28	80	-10	15	100	225	-150
32	85	-6	20	36	400	-120
36	75	-2	10	4	100	-20
$A = \boxed{38}$	59	0	-6	0	36	0
40	$B = \boxed{65}$	2	0	4	0	-17
39	48	1	-17	1	289	-60
42	50	4	-15	16	225	-33
41	54	3	-11	9	121	7
45	66	7	1	49	1	
		$\Sigma x = -13$	$\Sigma y = 2$	$\Sigma x^2 = 363$	$\Sigma y^2 = 1422$	$\Sigma xy = -453$

— (4)

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}\sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

$$= \frac{-453 - \frac{(-13)(2)}{10}}{\sqrt{\left(363 - \frac{169}{10}\right)\left(1422 - \frac{4}{10}\right)}}$$

$$= \frac{-453 + 2.6}{\sqrt{(363 - 16.9)(1422 - 0.4)}} = \frac{-450.4}{\sqrt{(346.1)(1421.6)}}$$

$$= -0.6421$$ — (2)

Q4 Let

a) $x + 6y = 6 \Rightarrow x = -6y + 6$. Reg line of x on y

$3x + 2y = 10 \Rightarrow y = \frac{1}{2}(-3x) + 5$ Reg line of y on x.

$\therefore\ b_{xy} = -6. \quad b_{yx} = -\frac{3}{2}.$

$b_{xy} \times b_{yx} = -6 \times -\frac{3}{2} = 9$

not possible. [since $r = \sqrt{b_{yx} \cdot b_{xy}}$.
$r = \pm 3$]

Hence,

$y = -\frac{1x}{6} + 1$

$b_{yx} = -\underline{\underline{1/6}}.$

$x = -\frac{2}{3}y + \frac{10}{3}.$

$b_{xy} = \underline{\underline{-\frac{2}{3}}}.$ —— 2

$b_{yx} \cdot b_{xy} = -\frac{1}{6} \times \frac{2}{3} \neq \frac{1}{9}$

$r^2 = \frac{1}{9} \Rightarrow \underline{\underline{r = -\frac{1}{3}}}.$

when $x = 12$

$y = -\frac{12}{6} + 1$

$y = -2 + 1 = \underline{\underline{-1}}.$ —— 2.

(b) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$ —— 1

$f(x) = x^3 - 5x + 3. \qquad f(1) = -1$

$f'(x) = 3x^2 - 5. \qquad f'(1) = -2$

First iteration

$x_0 = 1$

$x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - (-1/-2)$

$= 1 - 0.5 = \underline{\underline{0.5}}$

$f(0.5) = 0.625$

$f'(0.5) = -4.25$

Second iteration

$$x_2 = 0.5 - \left[\frac{0.625}{-4.25}\right]$$

$$= 0.5 + 0.1471$$

$$= \underline{0.6471}$$

$f(0.6471) = 0.0355.$

$f'(0.6471) = -3.7438$

Third iteration

$$x_3 = 0.6471 - \left[\frac{0.0355}{-3.7438}\right]$$

$$= \underline{0.6566} \quad \text{—— (2)}$$

$f(0.6566) = 0.000076.$

$f'(0.6566) = -3.7066$

Fourth iteration

$$x_4 = 0.6566 - \left[\frac{0.000076}{-3.7066}\right]$$

$$= \underline{0.6566}$$

Root $\underline{x = 0.6566}$ —— (1)

(c) $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7.$

$f_x = 3x^2 + 3y^2 - 6x.$ $\quad f_y = 6xy - 6y.$

$f_x = 0 \Rightarrow 3x^2 + 3y^2 - 6x = 0 \rightarrow (1)$ $\quad f_y = 0 \Rightarrow 6y(x-1) = 0.$

$y = 0$ (or) $x = 1$

Put $y = 0$ in (1)

$\Rightarrow 3x^2 - 6x = 0.$

$3x(x-2) = 0.$

$x = 0$ (or) $x = 2.$

$(0,0), (2,0)$

Put $x = 1$ in (1)

$3 + 3y^2 - 6 = 0.$

$3y^2 - 3 = 0.$

$y = \pm 1$

$\Rightarrow (1,1), (1,-1)$ — (2)

$r = f_{xx} = 6x - 6.$

$t = f_{yy} = 6x - 6.$

$s = f_{xy} = 6y.$

Points	r	t	s	$rt - s^2$	Interpretation	Value
$(0,0)$	-6 (<0)	-6	0	$36 > 0.$	maximum	$7.$
$(2,0)$	6 >0	6	0	$36 > 0$	minimum	3
$(1,1)$	0	0	6	$-36 < 0$	Saddle point	
$(1,-1)$	0	0	-6	$-36 < 0$	Saddle point	

— (4)

d) $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$|A - \lambda I| = 0.$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0.$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 2\left[2-\lambda-1\right] + 1\left[2-3+\lambda\right] = 0.$$

$$(2-\lambda)\left[6 - 2\lambda - 3\lambda + \lambda^2 - 2\right] - 2\left[1-\lambda\right] + \left[-1+\lambda\right] = 0.$$

$$(2-\lambda)\left[\lambda^2 - 5\lambda + 4\right] - 2\left[1-\lambda\right] - \left[1-\lambda\right] = 0.$$

$$(2-\lambda)(\lambda-4)(\lambda-1) + 2(\lambda-1) + (\lambda-1) = 0.$$

$$(\lambda-1)\left[2\lambda - \lambda^2 - 8 + 4\lambda + 3\right] = 0.$$

$$(\lambda-1)\left[-\lambda^2 + 6\lambda - 5\right] = 0.$$

$\lambda = 1, 1, 5.$ — 2

for $\underline{\underline{\lambda = 1}}$.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Put $x_3 = t_1$, $x_2 = t_2$.

$$x_1 + 2x_2 + x_3 = 0.$$

$$x_1 + 2t_2 + t_1 = 0.$$

$$x_1 = -2t_2 - t_1.$$

$$X = \begin{bmatrix} -t_1 - 2t_2 \\ 0 + t_2 \\ t_1 + 0 \end{bmatrix}$$

$$X_1 = t_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad X_2 = t_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad \text{—— (2)}$$

for $\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}\Bigg[$$

$$\sim \begin{bmatrix} 1 & -2 & 1 \\ -3 & 2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 \\ 0 & -4 & 4 \\ 0 & 4 & -4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_3 = t$$

$$-4x_2 = -4x_3 \Rightarrow x_2 = t$$

$$x_1 = +2x_2 - x_3 = 2t - t = \underline{\underline{t}}$$

$$X_3 = t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{— } 2$$