PILLAI COLLEGE OF ENG., NEW PANVEL (Autonomous) (1)

FE | SEM-I | COMP / IT / ECS | End Sem Exam | 2022 (Supplementary)

Q.P Code 210033

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1a P.T. $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8 = -2^8$

$\because 1+i\sqrt{3} = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$\therefore 1-i\sqrt{3} = 2\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]$

By D.M.V Th.

$(1+i\sqrt{3})^8 = 2^8\left[\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right]$

$(1-i\sqrt{3})^8 = 2^8\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right]$

$\therefore$ L.H.S $= (1+i\sqrt{3})^8 + (1-i\sqrt{3})^8$

$= 2^8\left[2.\cos\frac{8\pi}{3}\right]$

$= 2^9\left(-\frac{1}{2}\right) = -2^8$

1b $\tanh x = \frac{2}{3}$ (Given)

$\therefore \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2}{3}$

$\therefore \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{2}{3}$

$\therefore 3e^{2x} - 3 = 2e^{2x} + 1$

$\therefore e^{2x} = 5$

$\Rightarrow \boxed{x = \frac{1}{2}\log 5}$

$\therefore \boxed{\cosh 2x} = \frac{e^{2x} + e^{-2x}}{2} = \frac{5 + \frac{1}{5}}{2} = \boxed{\frac{13}{5}}$

1c

$$y = \frac{1}{x^2-5x+6} = \frac{1}{(x-3)(x-2)}$$

$$\therefore \frac{1}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$1 = A(x-2) + B(x-3)$$

Put $x = 2$ Put $x = 3$

$\boxed{B = -1}$ $\boxed{A = 1}$

$$\therefore y = \frac{1}{(x-3)} - \frac{1}{(x-2)}$$

$$\therefore \boxed{y_n = \frac{(-1)^n \lfloor n}{(x-3)^{n+1}} - \frac{(-1)^n \lfloor n}{(x-2)^{n+1}}}$$

1d

$$u = e^{xyz}$$

$$\frac{\partial u}{\partial z} = xy\, e^{xyz}$$

$$\frac{\partial^2 u}{\partial y\, \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = x\left[y\, e^{xyz}\, xz + e^{xyz}\right]$$

$$\therefore \frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = yz\left[x^2 e^{xyz}\, yz + e^{xyz}\, zx\right] + x\, e^{xyz}\, yz + e^{xyz}(1)$$

$$\boxed{\frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = e^{xyz}\left[1 + 3xyz + x^2 y^2 z^2\right]}$$

2a

$$17 \cosh x + 18 \sinh x = 1$$

$$17\left[\frac{e^x + e^{-x}}{2}\right] + 18\left[\frac{e^x - e^{-x}}{2}\right] = 1$$

$$\therefore \; 35 e^{2x} - 2e^x - 1 = 0$$

This eqn is quadratic in e^x

$$\therefore e^x = \frac{2 \pm \sqrt{4 + 140}}{70} = \frac{14}{70}, -\frac{10}{70}$$

For real value of x,

$$e^x = \frac{14}{70} = \frac{1}{5}$$

$$\therefore x = \log \frac{1}{5}$$

<u>2b</u> Expand $\sqrt{3}$ by taylor's Series

Let $f(x) = \sqrt{x}$

$\because f(x+h) = f(x) + h f'(x) + \frac{h^2}{\lfloor 2} f''(x) + \cdots\cdots$

$f(x) = x^{1/2}$, $f'(x) = \frac{1}{2} x^{-1/2}$, $f''(x) = -\frac{1}{4} x^{-3/2}$

$\therefore f(x+h) = x^{1/2} + h \cdot \frac{1}{2} x^{-1/2} + \frac{h^2}{\lfloor 2} \left(-\frac{1}{4}\right) x^{-3/2} + \cdots$

Put $x = 2$ & $h = 1$

$\therefore \sqrt{3} = 2^{1/2} + \frac{1}{2} 2^{-1/2} + \frac{1}{2}\left(-\frac{1}{4}\right) 2^{-3/2} + \cdots$

$= 1.7236$

$\therefore x = \log \frac{1}{5}$

2c $(2i \sin\theta)^7 = \left(x - \frac{1}{x}\right)^7$

$= \left(x^7 - \frac{1}{x^7}\right) - 7\left(x^5 - \frac{1}{x^5}\right) + 21\left(x^3 - \frac{1}{x^3}\right) - 35\left(x - \frac{1}{x}\right)$

$= (2i \sin 7\theta) - 7(2i \sin 5\theta) + 21(2i \sin 3\theta) - 35(2i \sin\theta)$

$\therefore \sin^7\theta = \frac{2i}{2 i^7}\left[\sin 7\theta - 7\sin 5\theta + 21 \sin 3\theta - 35 \sin\theta\right]$

$\therefore \boxed{\sin^7\theta = -\frac{1}{26}\left[\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta\right]}$

2d $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$

$A_{3\times3} = I_3 \, A \, I_3$

$\therefore \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

~~$R_3 \to R_3 + R_2$~~

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - 2C_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 + R_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C_3 \to C_3 - C_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

3a $y = f(x) = \sin x$ $\qquad \therefore y_0 = 0$

$\therefore y' = \cos x \qquad y'_0 = 1$

$y'' = -\sin x \qquad y''_0 = 0$

$y''' = -\cos x \qquad y'''_0 = -1$

$y^{IV} = \sin x \qquad y^{IV}_0 = 0$

$y^{V} = \cos x \qquad y^{V}_0 = 1$

$\vdots \qquad \vdots$

$\therefore$ By Maclaurin's Series

$$y = y_0 + x y'_0 + \frac{x^2}{\lfloor 2} y''_0 + \frac{x^3}{\lfloor 3} y'''_0 + \frac{x^4}{\lfloor 4} y^{IV}_0 + \frac{x^5}{\lfloor 5} y^{V}_0 + \cdots$$

$$= 0 + x(1) + 0 - \frac{x^3}{\lfloor 3} + 0 + \frac{x^5}{\lfloor 5} - \cdots$$

$$\boxed{\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} \cdots}$$

3b $\because u = 2x^2 - 3xy + y^2$

$\therefore u$ is a homogeneous function of degree 2.

$\therefore$ By Euler's Th.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$$

$$\therefore \boxed{x u_x + y u_y - 2u = 0}$$

3C

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ 0 & 1 & \lambda-1 & \mu-1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

for no Solⁿ

$\lambda - 3 = 0$ but $\mu - 10 \neq 0$

$\therefore \lambda = 3$ but $\mu \neq 10$

for unique Solⁿ

$\mu - 10 \neq 0$, λ may be any value

for infinite Solⁿ

$\mu - 10 = 0$, $\lambda - 3 = 0$

$\therefore \mu = 10$ & $\lambda = 3$

d) $f(x) = x^3 - 2x - 5$

$f(1) = -6 < 0$

$f(2) = -1 < 0$

$f(3) = 16 > 0$

$\therefore$ There is root between 2 and 3.

Now, $f'(x) = 3x^2 - 2$.

Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

iteration NO:-1, $x_0 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.1$$

iteration NO:-2 $x_1 = 2.1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{23.522}{11.23} = 2.0946.$$

iteration NO:-3 $\underline{x_2 = 2.0946}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \underline{2.0946.}$$

4) a) $z = \frac{1+4i}{4+i} \times \frac{4-i}{4-i}$

$= \frac{(1+4i)(4-i)}{4^2-i^2} = \frac{4-i+16i-i^2 4}{16+1}$

$= \frac{8+15i}{17}$

$z = \frac{8}{17} + \frac{15}{17} i$

Modulus $= \sqrt{\left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2}$

$= \sqrt{\frac{289}{17^2}} = 1$ $\boxed{\therefore |z| = 1}$

Argument $= \theta = \tan^{-1}\left(\frac{15/17}{8/17}\right)$

$\boxed{\theta = \tan^{-1}(15/8)}$

b) $e^{i\alpha} = i^{\beta}$

As,

$e^{i\theta} = \cos\theta + i\sin\theta$

& $i = \cos(\pi/2) + i\sin(\pi/2)$

$\therefore \cos\alpha + i\sin\alpha = \left(\cos(\pi/2) + i\sin(\pi/2)\right)^{\beta}$

$\therefore (\cos\alpha + i\sin\alpha)^{1/\beta} = \cos(\pi/2) + i\sin(\pi/2)$

$\therefore \cos(\alpha/\beta) + i\sin(\alpha/\beta) = \cos(\pi/2) + i\sin(\pi/2)$

(Useing De-Moiver's Thm).

$\cos(\alpha/\beta) + i\sin(\alpha/\beta) = \cos(2n\pi + \pi/2) + i\sin(2n\pi + \pi/2)$

$$\boxed{\therefore \alpha/\beta = 2n\pi + \pi/2}$$

c) $xy(3-x-y)$

We have, $f(x,y) = 3xy - x^2y - xy^2$

$f_x = 3y - 2xy - y^2$

$f_y = 3x - x^2 - 2xy$

$f_{xx} = -2y$

$f_{yy} = -2x$

$f_{xy} = 3 - 2x - 2y$

Now, $f_x = 0 = f_y$

$\therefore$ $y(3-2x-y) = 0$ & $x(3-x-2y) = 0$

$\therefore$ $y = 0$ & $y = 3-2x$

When, $\underline{y = 0}$

Then, $x(3-x-2y) = 0$ gives,

$x = 0$ & $x = 3$

$\therefore$ $(0,0), (3,0)$

when, $y = 3 - 2x$

Then, $x(3 - x - 2y) = 0$ gives

$x = 0$ or $x = 1$.

when, $x = 0$ then $y = 3$

$x = 1$ then $y = 1$.

$\therefore$ $(0,3)$, $(1,1)$ are stationary points.

Points	r	s	t	$rt - s^2$	
$(0,0)$	0	3	0	$-9 < 0$	saddle. point
$(3,0)$	0	-3	-6	$-9 < 0$	saddle point
$(0,3)$	-6	-3	0	$-9 < 0$	saddle point
$(1,1)$	-2	-1	-2	$3 > 0$	maxima

when, $y = 3-2x$

then, $x(3-x-2y)=0$ gives

$x=0$ or $x=1$.

when, $x=0$ then $y=3$

$x=1$ then $y=1$.

$\therefore$ $(0,3)$, $(1,1)$ are stationary points.

Points	r	s	t	$rt-s^2$	
$(0,0)$	0	3	0	$-9<0$	saddle. point
$(3,0)$	0	-3	-6	$-9<0$	saddle point
$(0,3)$	-6	-3	0	$-9<0$	saddle point
$(1,1)$	-2	-1	-2	$3>0$	maxima

d) $10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14.$

$x = \frac{1}{10}(12 - y - z)$

$y = \frac{1}{10}(13 - 2x - z)$

$z = \frac{1}{10}(14 - 2x - 2y)$

initial approximation (0, 0, 0).

<u>iteration No:- 1</u>

$x_1 = \frac{1}{10}(12) = 1.2$

$y_1 = \frac{1}{10}(13) = 1.3$

$z_1 = \frac{1}{10}(14) = 1.4$

<u>iteration No:- 2</u>

$x_2 = \frac{1}{10}[12 - (1.3) - (1.4)] = 0.93$

$y_2 = \frac{1}{10}[13 - 2(1.2) - (1.4)] = 0.92$

$z_2 = \frac{1}{10}[14 - 2(1.2) - 2(1.3)] = 0.9.$

iteration NO:- 3

$$x_3 = \frac{1}{10}[12-(0.92)-(0.9)] = 1.018$$

$$y_3 = \frac{1}{10}[13 - 2(0.93)-(0.9)] = 1.024$$

$$z_3 = \frac{1}{10}[14 - 2(0.93) - 2(0.92)] = 1.03$$

iteration NO:-4

$$x_4 = \frac{1}{10}[12 - (1.024) - (1.03)] = 0.9946$$

$$y_4 = \frac{1}{10}[13 - 2(1.018) - (1.03)] = 0.9934$$

$$z_4 = \frac{1}{10}[14 - 2(1.018) - 2(1.024)] = 0.9916$$

iteration NO:- 5

$$x_5 = \frac{1}{10}[12 - (0.9934) - (0.9916)] = 1.0015$$

$$y_5 = \frac{1}{10}[13 - 2(0.9946) - (0.9916)] = 1.0019$$

$$z_5 = \frac{1}{10}[14 - 2(0.9946) - 2(0.9934)] = 1.0024.$$